



2.1.5 Equality of vectors

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Introduction

- In this topic, we will
 - Define when two vectors are equal
 - Describe when two vectors are unequal
 - Visit the properties of equality





Equality of vectors

- Two n -dimensional real vectors are said to be equal if each of their entries are equal as real numbers
 - That is $\mathbf{u}, \mathbf{v} \in \mathbf{R}^n$ are equal (written $\mathbf{u} = \mathbf{v}$) if and only if

$$u_1 = v_1, u_2 = v_2, \dots, \text{ and } u_n = v_n$$

- For example,
$$\begin{pmatrix} \frac{1}{2} \\ -\frac{3}{4} \\ \frac{1}{4} \end{pmatrix} = \begin{pmatrix} 0.5 \\ -0.75 \\ 0.25 \end{pmatrix}$$





Unequal vectors

- Even if vectors differ in a single entry, they are unequal

$$\begin{pmatrix} 0.65 \\ -0.09 \\ 0.93 \\ -1.53 \end{pmatrix} \neq \begin{pmatrix} 0.65 \\ -0.09 \\ 0.98 \\ -1.53 \end{pmatrix} \quad \begin{pmatrix} 0.65 \\ -0.09 \\ 0.93 \\ -1.53 \end{pmatrix} \neq \begin{pmatrix} 0.65 \\ -0.09 \\ -1.53 \\ 0.93 \end{pmatrix}$$

- In some cases, two vectors may be *close enough*
 - To be defined precisely once we calculate the distance between two vectors

$$\begin{pmatrix} 0.58238975813 \\ -0.01509398133 \\ 1.91738584023 \end{pmatrix} \approx \begin{pmatrix} 0.58238969394 \\ -0.01509415838 \\ 1.91738602931 \end{pmatrix}$$





Properties of equality

- The properties of equality include:
 - Every vector is equal to itself: $\mathbf{u} = \mathbf{u}$ *reflexive*
 - If $\mathbf{u} = \mathbf{v}$, then $\mathbf{v} = \mathbf{u}$ *symmetry*
 - If $\mathbf{u} = \mathbf{v}$ and $\mathbf{v} = \mathbf{w}$, then $\mathbf{u} = \mathbf{w}$ *transitive*
 - If $\mathbf{u} = \mathbf{v}$, then any statement about $\mathbf{u}$ is true for $\mathbf{v}$ and vice versa





Different dimensions

- We do not compare vectors that do not have the same dimension
 - It's like comparing apples and oranges
 - We focus on one dimension at a time





Summary

- Following this topic, you now
 - Understand the concept of vector equality
 - Understand we never compare vectors of different dimensions
 - Have been exposed to the properties of equality





References

- [1] [https://en.wikipedia.org/wiki/Equality_\(mathematics\)](https://en.wikipedia.org/wiki/Equality_(mathematics))





Acknowledgments

None so far.





Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using Consolas.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

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for more information.





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